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Realism, Locality and Einstein

Classifying and understanding realistic views of Quantum Mechanics

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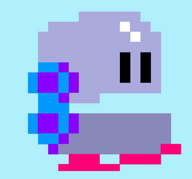


Formalizing realistic models of Quantum Mechanics

- A **realistic** view of reality takes there to be some set of *ontic states*, $\lambda \in \Lambda$ describing a system.

- Quantum Mechanics (QM) is then seen to be a 'coarse-grained' view of the dynamical evolution of these ontic states.

- QM talks about preparations and measurements – using density operators and POVMs

How does a realistic model deal with preparations and measurements?

- A preparation described by density operator ρ is taken to give only *incomplete* knowledge of which λ is prepared, defining an ***epistemic distribution***, $\mu(\lambda|\rho)$.

- Upon interacting with a measuring device, certain λ will trigger certain measurement outcomes. A POVM $\{E_k\}_k$ is associated with a set of ***indicator functions***, $\xi_{E_k}(\lambda)$, which associate a probability for obtaining a measurement outcome corresponding to some POVM element, E_k, with each ontic state.

Reproducing Quantum Mechanical statistics

- A **realistic** model must reproduce quantum mechanical statistics when seen from a 'coarse-grained' perspective. In the above formalism, the probability of obtaining an outcome E_k in some measurement, given that a preparation procedure associated with QM density operator ρ is performed beforehand, is:

$$\int_\Lambda d\lambda \mu(\lambda|\rho)\xi_{E_k}(\lambda)$$

- Thus agreement with QM requires:

$$\int_\Lambda d\lambda \mu(\lambda|\rho)\xi_{E_k}(\lambda) = tr\left(\rho E_k\right)$$

References:

[1] N. Harrigan, R. W. Spekkens, In preparation.

[2] Einstein to Schrödinger, 19 June 1935, extracts translated in D. Howard, "Einstein on Locality and Separability", Studies in History and Philosophy of Science **16**, 171-201 (1985).

Why consider realistic models?

There are numerous results showing that a completely classical **realistic** theory cannot reproduce the predictions of QM. By studying how one must perturb a classical **realistic** view of reality we hope to identify the fundamentally strange features of QM. These provide a focal point for assessing any potential interpretation of the quantum formalism. In particular, we would hope any successful **non-realistic** interpretation to provide some intuition as to why and how these features occur. In other work we follow this approach by investigating in what ways ***contextuality*** must perturb a classical **realistic** theories describing quantum mechanics. Here we provide a classification of **realistic** theories and demonstrate how one can easily prove non-locality for all but one of these classes **[1]**. Interestingly, the proof we give was first published by Einstein, 28 Years before Bell's famous theorem demonstrated non-locality for *any* **realistic** theory.

$$\boxed{\text{QM}} = \boxed{\text{CM}} + \boxed{\textbf{?}}$$

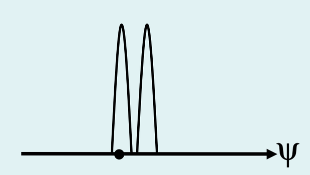
The motivation for our study is to understand precisely how classical mechanics fails to describe reality in light of quantum mechanics

Relating ψ to reality: Classifying realistic theories

Simply associating $\rho=|\psi\rangle\langle\psi|$ with a distribution $\mu_\psi(\lambda)$ does not specify entirely how ψ relates to the ontic states $\lambda \in \Lambda$ (i.e. to reality). We introduce three exhaustive classifications for how a **realistic** theory relates a systems projective Hilbert space, *PH,* to the underlying reality Λ.

- **ψ – complete theories:**

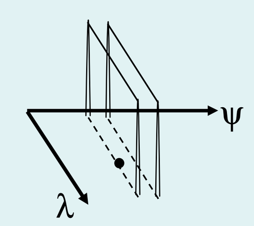
In these theories ψ is taken to be the ***complete*** state of a system, so that knowledge of ψ is the most that one can ever hope to know about a system and $PH=\Lambda$. This is the kind of relation implicit in what most would call the orthodox interpretation.



*In a ψ complete theory, the **epistemic distributions** associated with quantum states are delta functions, providing a one-to-one mapping between PH and Λ.*

- **ψ – supplemented theories:**

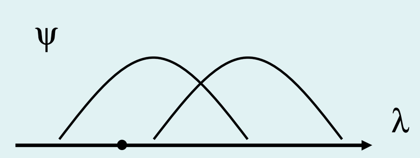
In a ψ-supplemented theory, although quantum states are taken to provide direct descriptions of reality (as in the case of ψ-complete theories), they are not sufficient to completely determine the real state of the system. A complete description of reality also requires specifying some additional ontic states λ. Thus for ψ-supplemented theories we have that $PH \subset \Lambda$.



In a ψ-supplemented theory, ψ gives only an incomplete description of reality, which is completely specified by the pair (ψ,λ). A given ψ in general only defines a distribution over the λ (shown in above figure by another axis).

- **ψ – epistemic theories:**

In ψ-epistemic theories, varying the quantum state does not neccesarily lead to a variation in the ontic states describing a system. Rather ψ only defines a probability distribution over Λ. A more rigorous definition would state that a theory is ψ-epistemic if we can find quantum states ψ and ϕ such that $\mu(\lambda|\psi)\mu(\lambda|\phi)>0$ for some $\lambda \in \Lambda$ i.e. if there are some ***epistemic distributions*** which overlap - so that one cannot construct a one-to-one mapping between quantum states and *sets* of ontic states.



In a ψ-epistemic theory the quantum state defines only a probability distribution over the ontic states $\lambda \in \Lambda$, which encodes our lack of knowledge. A change in ψ does not necessarily result in a variation of the real state of affairs.

Showing Non-locality without a Bell inequality

It is possible to show that any model that is ψ-complete or ψ-supplemented must be non-local, simply because of a phenomenon known as steering, which is exhibited by entangled quantum states.

To demonstrate steering, consider Alice and Bob to each share a subsystem of the following entangled quantum state,

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

By choosing to measure her subsystem in either the basis $\{|0\rangle,|1\rangle\}$ or $\{|+\rangle,|-\rangle\}$, Alice can choose whether Bob's subsystem will consequently be described by either

$$\rho_B^{01} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \qquad \text{or} \qquad \rho_B^{\pm} = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

Alice is therefore able to *'steer'* the state of Bob's subsystem to lie in either the set $\{|0\rangle,|1\rangle\}$ or the set $\{|+\rangle,|-\rangle\}$.

A **realistic** model associates ***epistemic distributions*** $\mu(\lambda|\rho^{01}_B)$ and $\mu(\lambda|\rho^{\pm}_B)$ with ρ^{01}_B and $\rho^{\pm}_B$ respectively. It can be shown operationally that these distributions must follow analogous convex decompositions to those shown above for ρ^{01}_B and $\rho^{\pm}_B$,

$$\mu(\lambda|\rho_B^{01}) = \frac{1}{2}\mu(\lambda|\,|0\rangle) + \frac{1}{2}\mu(\lambda|\,|1\rangle) \quad \text{and} \quad \mu(\lambda|\rho_B^{\pm}) = \frac{1}{2}\mu(\lambda|\,|+\rangle) + \frac{1}{2}\mu(\lambda|\,|-\rangle)$$

Locality requires that Alice's choice of measurement basis should not alter the probability distribution describing Bob's system, so that we have $\mu(\lambda|\rho^{01}_B)=\mu(\lambda|\rho^{\pm}_B)=\mu(\lambda|I/2)$. Multiplying together the expressions for $\mu(\lambda|\rho^{01}_B)$ and $\mu(\lambda|\rho^{\pm}_B)$ thus gives,

$$4\mu^2(\lambda|I/2) = \mu(\lambda|+)\mu(\lambda|0) + \mu(\lambda|+)\mu(\lambda|1) + \mu(\lambda|-)\mu(\lambda|0) + \mu(\lambda|-)\mu(\lambda|1)$$

For any λ in the support of $\mu(\lambda|I/2)$ we must have $4\mu^2(\lambda|I/2)>0$ and therefore

$$\mu(\lambda|+)\mu(\lambda|0) > 0 \quad \text{or} \quad \mu(\lambda|+)\mu(\lambda|1) > 0 \quad \text{or} \quad \mu(\lambda|-)\mu(\lambda|0) > 0 \quad \text{or} \quad \mu(\lambda|-)\mu(\lambda|1) > 0$$

This thus ensures that the theory is ψ-epistemic, showing that only ψ-epistemic **realistic** theories can reproduce the quantum mechanical steering phenomenon in a local fashion.

Non-locality and Einstein

- In 1935 Einstein mentioned in his correspondence an argument for incompleteness *differing markedly from the one published in the famous EPR paper earlier during the same year* **[2]**.

- The argument operates by noting that Alice's ability to *'steer'* the state of Bob's subsystem (as outlined above) can only be explained in a *local* theory by associating more than one quantum state (dependent on Alice's choice of steering basis) with a single ontic state of Bob's subsystem.

- Einstein argument thus concludes that any local **realistic** theory cannot associate quantum states with ontic states in a one-to-one fashion. Therefore the argument shows that any local **realistic** theory cannot be ψ-complete, but instead must be either ψ-supplemented or ψ-epistemic.

- However Einstein's argument can be trivially adapted (by strengthening an initial assumption) to bring it precisely into the form of the non-locality proof that we have previously given, showing that only ψ-epistemic **realistic** theories are compatible with locality **[1]**.

- Historically speaking, this provides evidence that Einstein's envisaged completion of quantum mechanics was in the form of what we would now classify as a ψ-epistemic **realistic** theory – a theory wherein there is not always a link between varying ψ and varying the realistic state of the system.